XXIV. *The principal Properties of the Engine for turning Ovals in Wood or Metal, and of the Inſtrument for drawing Ovals upon Paper, demonſtrated. By the Rev. Mr.* Ludlam, *Vicar of* Norton, *near* Leiceſter; *communicated by the Aſtronomer Royal.*

Read May 4, 1780.

THE inſtrument for drawing ovals upon paper or board is ſo common, that a particular deſcription of it is needleſs. It is much in uſe among the joiners, and called by them *the trammels*. One part of it conſiſts of a croſs with two grooves at right angles: the other is a beam carrying two pins which ſlide in thoſe grooves, and alſo the deſcribing pencil; we ſhall diſtinguiſh theſe two parts by the names of the *croſs* and the *beam*.

It is very well known, that all the engines for turning ovals are conſtructed on the ſame principles with the trammels; the only difference is, that in the trammels the board is at reſt, and the pencil moves upon it; in the turning engine, the tool (which ſupplies the place of the pencil) is at reſt, and the board moves againſt it.

Let

Let A*a* and B*b* (fig. 1.) be two indefinite lines, interfecting each other at right angles in C. Let LSM be the beam, or a rigid right line, in which affume two fixed points L and S at pleafure. If the fixed point L be kept always fliding upon the line BC*b*, and the other point S always fliding upon the line AC*a*; I fay then, that any point M in the line LS, or that line produced, will defcribe an ellipfe.

Bifect LS in E, and through C and E draw the indefinite right line CEH. Upon LS as a diameter with the center E defcribe a femi-circle, and becaufe LCS is a right angle, it will pafs through C, and EC=EL. Through M draw MPH perpendicular to AC meeting CE produced in H; and becaufe MH is parallel to CL, the triangles MEH and CEL are fimilar, and HE=ME, and HE+EC=ME+EL, or CH=LM. The point H therefore always falls in the circumference of the circle HAD*a* defcribed with the center C and radius CH=LM. Now the fimilar triangles CHP and SMP give CH : SM :: PH : PM. But when L arrives at C, then LM (=CH) coincides with CA; and when S arrives at C, then SM coincides with CB; therefore CA : CB :: PH : PM, and $CA^2 : CB^2 :: PH^2 : PM^2$, or $CA^2 : CB^2 :: AP \times Pa : PM^2$, which is the property of an ellipfe, whofe firft femi-axe is CA or LM, and fecond femi-axe is CB=SM.

 Produce

Produce PM till it meets the circle in N, and draw the radius CN; then PH=PN and CA : CB :: PN : PM. Again, becaufe PCH=PCN, therefore NCD=ECL=ELC and CN is parallel to LM, and CL=NM. Draw M*p* perpendicular to B*b*, cutting CN in *n*, and for the like reafon C*n*=SM=CB, and CS=M*n*. While the point M defcribes an oval, the point E defcribes a circle whofe center is C and radius CE=$\frac{1}{2}$SL.

To the ruler MEL (fig. 2. and 3.) fix another ruler or right line *m*EK paffing through E, fo that the ruler *m*EK may be carried about by the ruler MEL, keeping the angle ME*m* between the two rulers invariable. On *m*EK take EV=EK, and each = ES or EL, I fay, the point V will defcribe a right line *a*VC*α* paffing through C, and making an angle *a*CS with CA, equal to half ME*m* the angle made by the two rulers; the point K will alfo defcribe a right line *b*KC*β* paffing through C, and making an angle *b*CL, with CL, alfo equal to half ME*m*.

On the center E (fig. 2. and 3.) and with the radius EC defcribe a circle and it will pafs through the points S, V, C, L, K; draw the lines VC and KC, and the angles SEV, and SCV, both ftand on the fame arch SV; the former at the center E, the latter at the circumference C; therefore the former is double the latter. In like manner the angles KEL and KCL both ftand on the fame arch KL, the former at the center, the latter at the circumference; therefore the former is double the latter. Now as this holds in every poſition of the rulers during their joint motion, it is manifeſt, the points V and K will each deſcribe right lines, namely, *a*C*α* and *b*C*β*, paſſing through C and making the angles *a*CA and *b*CL (=BC*β*) each equal to half ME*m*.

Hence the lines *a*C*α* and *b*C*β*, traced by the points V and K are at right angles, and the ruler *m*VEK moves exactly in the ſame manner as if it was guided by the the points V and K ſliding on the lines *a*C*α* and *b*C*β*, at right angles to each other; juſt in the ſame manner as the ruler MSL is guided by the points S and L, ſliding on the lines AC and BC. Therefore if any point *m* be aſſumed in the line KV*m* as a deſcribing point, the figure deſcribed will be an ellipſe, the poſition of whoſe principal axes are the lines *a*C*α* and *b*C*β*; the center of the ellipſe being ſtill in C as before. If *m*K is taken equal to ML, the ellipſe thus deſcribed by the point *m* will be the ſame with that deſcribed by the point M, only in another poſition: its greater ſemi-axis *a*C making an angle with AC, the greater ſemi-axis of the former ellipſe, equal to half ME*m*, the angle which the rulers or lines ME and *m*E make with each other.

Scholium. This propoſition is demonſtrated in SCHOOTEN'S *Exercitationes,* &c. p. 305.; but he makes twelve caſes of it: had he made uſe of the 20th of the 3d El. they might have been all comprehended under one.

In the turning of ovals, the top of the *reſt* which ſupports the tool is always made to paſs through S and L (fig. 1.) the two centers round which the oval engine turns; and in this caſe the ruler or line MSEL repreſents the top of the *reſt*. If the tool be held on any part of the *reſt* between the work-man, and the neareſt center as at M, an oval will be turned having its longer axis A*a* (in one poſition of the work) coinciding with the top of the *reſt*. As the tool is removed towards S, the oval will grow narrower and at S become a right line. Beyond S towards E it will grow rounder, and at E become a circle; beyond E it will grow narrower, and at L become a right line at right angles to the right line deſcribed when the tool was at S. If the tool be removed beyond L, it will deſcribe an oval again, whoſe longer axis is at right angles to the longer axis of the oval firſt deſcribed when the tool was at M. It may be very convenient to mark the points S and L and alſo their middle point E on the top or face of the *reſt* that ſupports the tool. If any thing be interpoſed between the tool and the top of the *reſt* ſo as to raiſe the tool above the line paſſing through the centers

S and

S and L, an oval will yet be defcribed, whofe center will be the fame with that of the oval firft defcribed when the tool was at M; but its principal axis will crofs the principal axis of that oval (fig. 2. and 3.). Draw right lines both from M the old place of the tool, and from *m* the new place of the tool, to the point E marked on the *reft*. Half the angle which thefe two lines make with each other will be the angle which the principal axis of the new oval makes with the principal axis of the old one.

It is well known, that when the oval engine is fet in order for working, there is a part which flides back, and is then fixed, which feparates the two centers of motion and gives the eccentricity; for the difference between the firft and fecond femi-axes will be juft as much as the centers are thus feparated: call the diftance between the two centers E; let now the tool be fixed in any place, upon, above, or below the *reft*; call *m* E the diftance of the tool from the middle point between the centers (marked E on the *reft)* D; and the greater femi-axis of the oval fo defcribed will be $D + \frac{1}{2}E$, and the leffer femi-axis $D - \frac{1}{2}E$; and thus both the form and pofition of the oval will be known. All workmen know the tool muft never be raifed above the place where it was at firft held, and we

 fee

ſee the reaſon; it would deſtroy the oval firſt begun to be turned, and form a new one in a different poſition.

But there is another difficulty in turning ovals, eſpecially ſuch as have mouldings, as picture-frames, &c. The tool generally has all the mouldings formed upon it: now if it be laid flat upon the *reſt*, and the engine ſet to work; the mouldings will in ſome places croſs the plane of the tool (or the top of the *reſt)* at right angles (as in turning circles), in other places obliquely. This will make the ſeveral members of the mouldings *leaner* or ſmaller in one part of the work than another. Nor will the caſe be altered if the mouldings be turned ſeparately. Analogous to this, when an oval is drawn by the trammels, the line deſcribed by the pencil will not, as in a circle, be always at right angles to the beam of the trammels. The oval line ſo drawn will be at right angles to the deſcribing beam, only at the extremity of the two principal axes where the beam coincides with thoſe axes; in all other places the oval line and beam make an oblique angle. It may be proper therefore to enquire how much this angle deviates from a right angle. This we ſhall call the angle of *deviation*.

All things as in fig. 1. draw the tangents TM and TN, to the point M in the ellipſe and the point N in the circle correſponding to each other; and from the nature of the ellipſe

ellipse these tangents will meet each other in the axis CA produced. Draw MG perpendicular to TM, and GMS will be the angle of deviation sought. I say, the angle MTN, between the tangents to corresponding points in the ellipse and circumscribing circle, is equal to the angle of deviation GMS.

For because TNC is a right-angled triangle, and NP perpendicular to TC; therefore TNP=NCP=MSP, that is (in the triangles MTN and GMS) the angles TNM and MSG are equal. In like manner, because TMG is a right-angled triangle and MP perpendicular to TG, therefore TMP= MGP, and (in the triangle MTN and GMS) the angles TMN and MGS are equal; therefore in the same triangles, the remaining angles MTN and GMS are also equal.

To compute the angle MTN, we have by trigonometry $TP^2 + PM \times PN : TP :: MN : \tan. MTN$, radius being unity. Call now $CA=t$, $CB=c$, $CP=x$, $PM=y$, $CA-CB$ (or $t-c$) $=d$, and we have $PN=\sqrt{tt-xx}$; also $CD : CB :: PN : PM = \sqrt{tt-xx} \times \frac{c}{t}$, whence $PM \times PN = \overline{tt-xx} \times \frac{c}{t}$. Again $CP : PN :: PN : PT$, whence $TP = \frac{tt-xx}{x}$. Lastly, $CD : BD :: PN : MN = \sqrt{tt-xx} \times \frac{d}{t}$; whence the tangent of MTN the angle sought is $\frac{dx\sqrt{tt-xx}}{ttt-dxx}$: and this is a *maximum* when $\frac{ttt}{2t-d}$, or $\frac{ttt}{t+c} = xx$, or when $\frac{ccc}{t+c} = yy$, or when CP and PM have such a proportion that $CP^2 : PM^2 :: CA^3 : CB^3$.

Let

Let AMB*ab* (fig. 4.) be an ellipſe whoſe center is C; draw the circumſcribed and inſcribed circles as before; the former cutting the ſecond axis produced in D, the latter cutting the firſt axis in *d*, and the ſecond axis in *b*. On D*b* as a diameter deſcribe a circle cutting the firſt axis A*a* in Q, draw DQ and *b*Q. Set off CR=DQ, join DR, and draw DP at right angles cutting the firſt axis in P, draw PM an ordinate to that axis, and M will be the point in the oval line where the angle of deviation is greateſt. Otherwiſe, upon C*b* produced ſet off C*r*=*b*Q, join *dr*, and draw *dp* at right angles cutting the ſecond axis in *p*: draw *p*M an ordinate to that axis, and M will be the point where the angle of deviation is greateſt.

At the maximum (when $xx=\frac{ttt}{t+c}$) $PN^2=\frac{ttc}{t+c}$, $PM^2=\frac{ccc}{t+c}$, and $TP^2=\frac{tcc}{t+c}$: whence $TP^2=PM\times PN$. Alſo, PN, PM, TP, are to each other as CA, CB, and $\sqrt{CA\times CB}$, reſpectively. Therefore, $\frac{CA-CB}{\sqrt{Aa\times Bb}}$ is the tangent of MTN, radius being unity. Alſo $\sqrt{\frac{CA}{CB}}$ is the tangent of NTP; and MTP is the complement of NTP: therefore MTN is twice the exceſs of NTP, above 45°.

ADDENDA.

At the end of page 386 *add,*

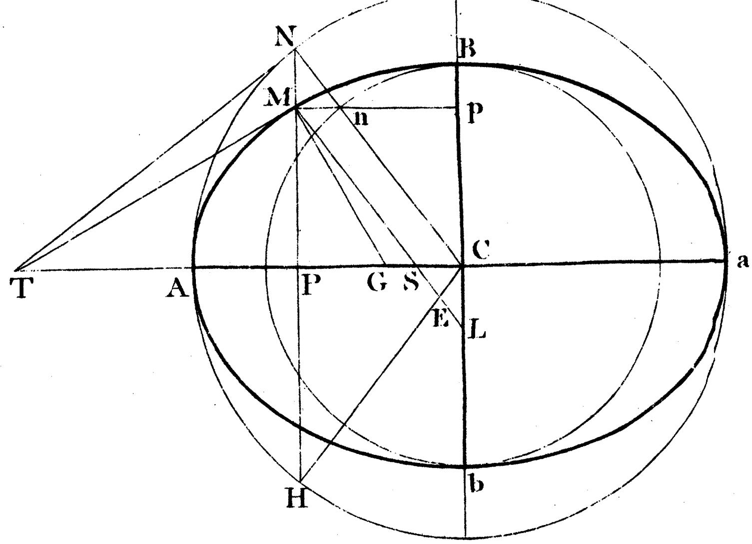
In fig. 4. draw a circle through the points N, M, T; and at the maximum where $TP^2 = PM \times NP$ this circle will touch CA produced in T. From E the center of this circle draw EF perpendicular to NM, alſo the radii EN and EM; and FN is the ſine of NEF, or half NEM, or of its equal MTN, to the radius EN. But $EN = ET = PF = \frac{PN+PM}{2}$, and $FN = \frac{PN-PM}{2}$. Therefore PN + PM is to PN − PM, or CD + CB is to CD − CB, or CA + CB is to CA − CB, as radius is to the ſine of the greateſt angle of deviation, which is therefore equal to $\frac{CA-CB}{CA+CB}$, radius being unity.

ERRATA to VOL. LXX.

Page 6, *line antepenult. read* be nearly mathematically.
6, *l. penult. dele* yet.
7, *l.* 13, *dele* ſection ABC, or
7, *l.* 18, *at the end of the line add* very nearly.
394, *l.* 15, *tranſpoſe* general equation *to the beginning of the line above.*
402, *l.* 6, 7, 8, *for* 9143 *r.* 9443.
405, *l.* 7, *for the laſt,* $1+\sqrt{-3}$ *r.* $1-\sqrt{-3}$.
405, *l.* 11, *for the laſt* $-\frac{\sqrt{-3}}{2}$ *r.* $+\frac{\sqrt{-3}}{2}$.
443, *end of the 1ſt line, for* and x *r.* and X.
548, *l.* 10, *for* circumſtances *r.* and which are.

*** There are FIFTEEN Plates in this Volume.

Fig. 1.



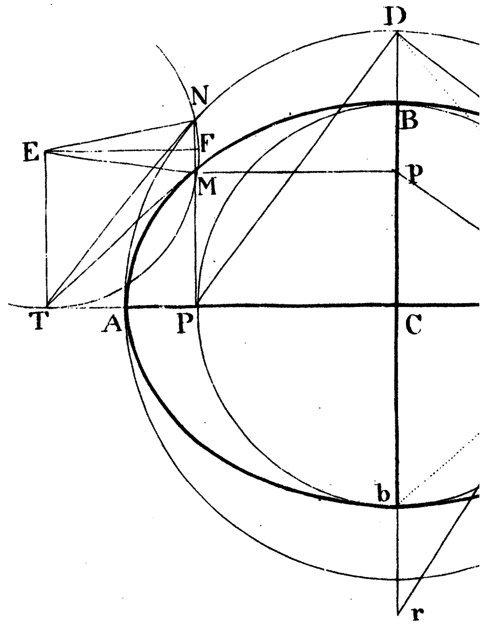


Fig. 2.

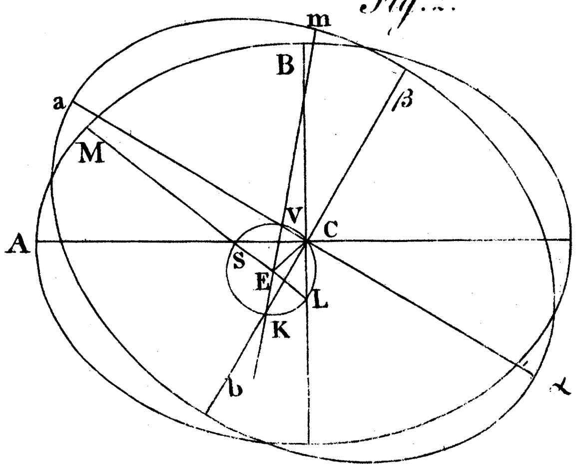
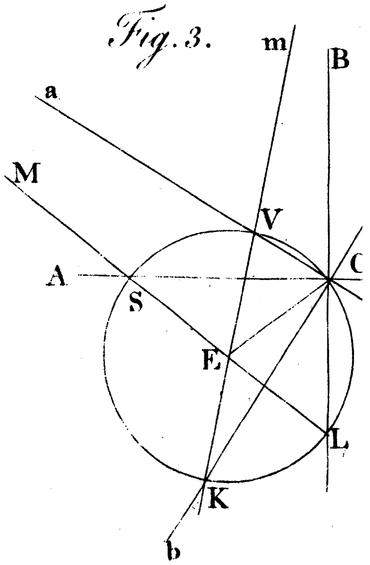


Fig. 3.



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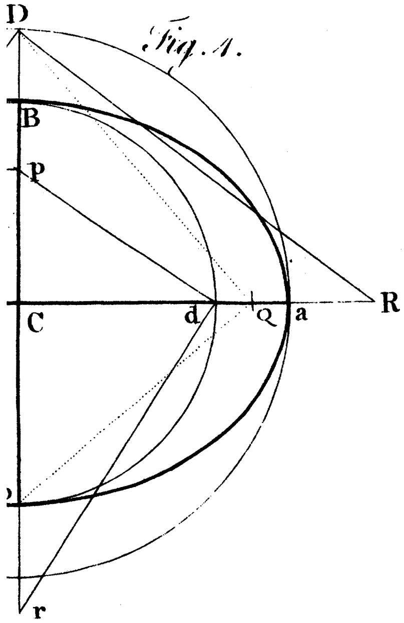

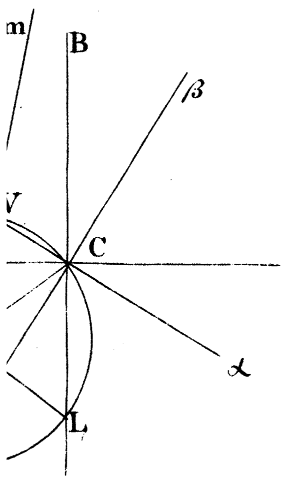

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Fig. 1.

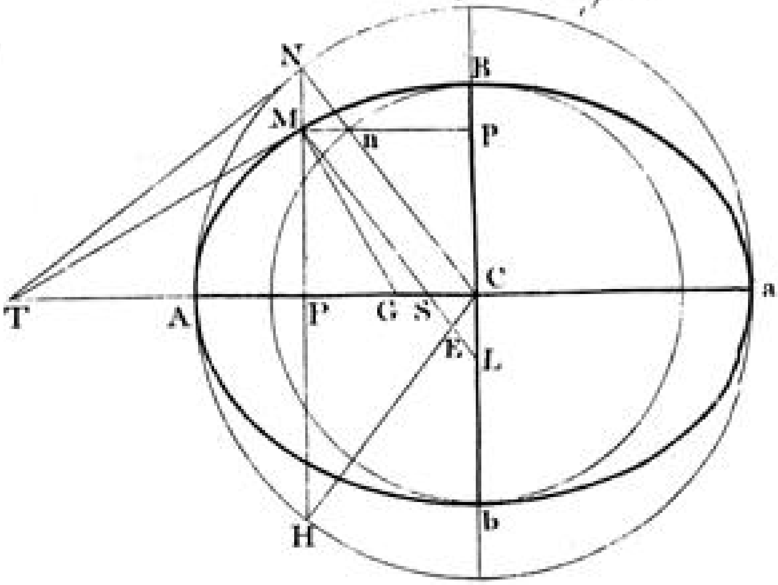

Fig. 4.

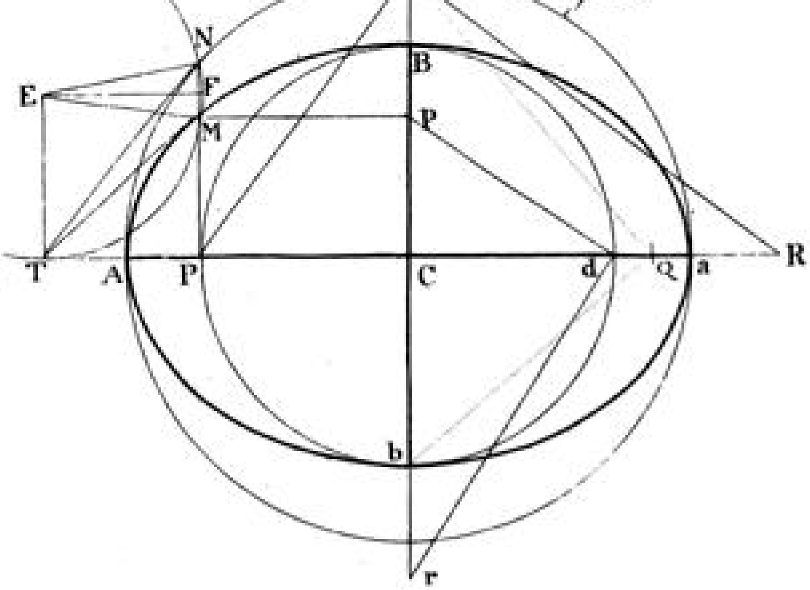

Fig. 2.

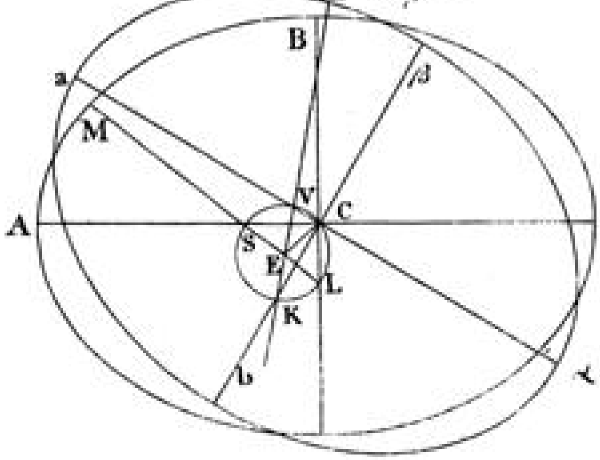

Fig. 3.